

# Comparative Study of the Performance of Two way and One Way DF Relaying with Co-Channel Interference and Channel Estimation Error

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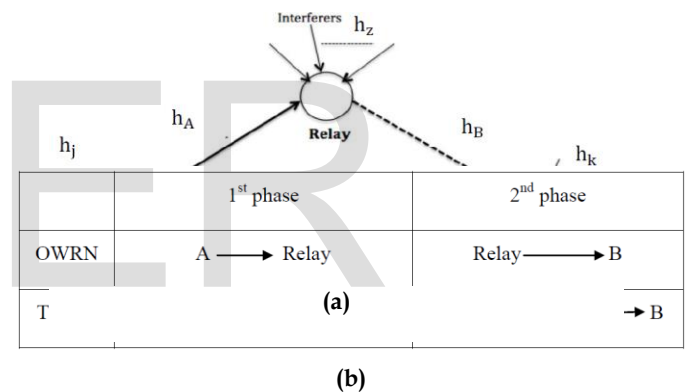
**Abstract**— It is known that cooperative communications enhances the bit error rate performance of wireless communication systems. In this letter, we show that cooperative communications also reduce the effect of channel estimation error and co-channel interference. A metric called signal-to-noise gap ratio is used to measure the amount of this reduction in decode and forward two-way relay network (TWRN) and one way relay network (OWRN) compared with the direct link between source and destination. The effect of different parameters on the ability of TWRN and OWRN to reduce the effect of channel estimation error and co-channel interference is studied. It is shown that OWRN achieves more reduction on the effect of channel estimation error and co-channel interference than TWRN. Furthermore, more reduction can be achieved if the number of relays increased.

**Index Terms**— Two way relaying, One way relaying, Decode and Forward.

## 1 INTRODUCTION

Recently gained large interest in the research community [1]. It has been shown that cooperative communications have the advantage of improving the capacity and the reliability of the communication, save the battery consumption for extending network lifetime and expanding the transmission coverage area [2]. There are two types of networks that use the relay nodes, which are OWRN and TWRN. In OWRN, the source sends the data to the relay in one time slot then the relay forwards the data to the destination node in another time slot. The disadvantage of OWRN is that it has a capacity loss due to the one way relaying. The TWRN solves this problem by letting the two source nodes send their data simultaneously to the relay node in one time slot, then in the second slot the relay broadcasts the data to the nodes [3].

Successful communication depends on how accurately the receiver detects the transmitted signal. Perfect channel detection might be possible in the absence of noise and other effects. There are problems existing in wireless networks such as channel estimation error and co-channel interference. One of the possibilities that cause the channel estimation error is the Doppler shift that can cause a dramatic degradation in the performance of the wireless communication. It was shown in [4] that the channel estimation error results in lower average



**Figure 1: System model (a) Relay network with co-channel interference (b) OWRN and TWRN protocols**

signal-to-noise ratio (SNR) and higher average error rate. Beside the channel estimation error problem, there is the co-channel interference problem which arises in the network as users of different neighboring cells transmit their data simultaneously over the same channel. Co-channel interference causes a decrease in signal to interference plus noise ratio (SINR), which leads to degradation in the performance.

## 2 SYSTEM AND CHANNEL MODELS

Consider a relay network, consisting of two terminals A and B (destination) and a relay R, all of them are equipped with a single antenna as shown in Fig. 1(a). We assume that the relay and the terminals suffer from M co-channel interferers. The fading channel coefficients from source A to the relay and from source B to the relay links are denoted by  $h_A$  and  $h_B$  respectively. The channels between interferers and source A, source B, and relay are denoted by  $h_j$ ,  $h_k$ , and  $h_z$  respectively. The channel coefficients  $h_A$ ,  $h_B$ ,  $h_j$ ,  $h_k$  and,  $h_z$  are modeled as zero mean complex Gaussian random variables with variances  $\delta_A^2$ ,  $\delta_B^2$ ,  $\delta_j^2$ ,  $\delta_k^2$  and  $\delta_z^2$  respectively. The additive noise is as-

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sumed to be white Gaussian noise with zero mean and unit variance. The OWRN and TWRN protocols spend two time phases as shown in Fig. 1(b). In essence, source A delivers its data to source B via the relay in the OWRN protocol. Without loss of generality, we assume that source A and B have the same transmitted power  $P_0$  and the relay has a transmitted power  $P_1$ .

### 3 SIGNAL TO NOISE GAP RATIO OF DECODE AND FORWARD RELAY NETWORK

#### 3.1 Signal to noise gap ratio of two way relaying

In this sub-section we derive the average SNR-gap ratio which represents the loss in SNR due to channel estimation error and co-channel interference [9]. In phase 1, the two sources send their information messages at the same time to the relay node. The received signal at the relay from source A and source B is given by

$$y_r = \sqrt{P_0} (h_A + e_A) x_A + \sqrt{P_0} (h_B + e_B) x_B + \sum_{k=1}^M P_k \delta_k^2 + \sqrt{N_0} (n_A + n_B) \quad (1)$$

where  $P_0$  is the source transmitted power,  $x_A$  and  $x_B$  are the transmitted signals from source nodes A and B respectively. In (1), the terms  $e_A$  and  $e_B$  are the channel estimation errors of channels  $h_A$  and  $h_B$  respectively and are modeled as zero mean complex Gaussian random variables with same variance  $\alpha$ . While the terms  $n_A$  and  $n_B$  are the noise received from source A and B respectively and are modeled as zero mean complex Gaussian random variables with unit variance. The second term in the above equations represents the co-channel interference signal received from M interferers. For sufficiently large number of interferers M and according to the central limit theorem, the interference terms  $\sum_{j=1}^M \sqrt{P_j} h_j x_j$  and  $\sum_{k=1}^M \sqrt{P_k} h_k x_k$  can be modeled as zero mean complex Gaussian random variables with variances  $\sum_{j=1}^M P_j \delta_j^2$  and  $\sum_{k=1}^M P_k \delta_k^2$  respectively. Moreover, for a constant modulus transmitted signals  $x_A$  and  $x_B$  the additional self-noise terms  $(\sqrt{P_0} x_A e_A)$  and  $(\sqrt{P_0} x_B e_B)$  are zero mean complex Gaussian random variables with variance  $\epsilon P_0$  [9]. Then (1), can be written as

$$y_r = \sqrt{P_0} (h_A x_A + h_B x_B) + \sqrt{\sum_{k=1}^M P_k \delta_k^2 + \epsilon P_0 + N_0} n_r \quad (2)$$

where  $n_r = n_A + n_B$ . In the second phase, the relay jointly decodes the received signals from the two source nodes to obtain  $\hat{x}$  where  $\hat{x} = x_A + x_B + e_t$ ; where  $e_t$  is the error of decoding. The relay then broadcasts the obtained signal  $\hat{x}$  to the two sources A and B. The transmitted signals from the relay to the source A and B are given respectively by

$$y_A = \sqrt{P_1} h_A (x_A + x_B) + \sqrt{P_1} h_A e_t + \sqrt{\epsilon P_1 + \sum_{j=1}^M P_j \delta_j^2 + N_0} n_A$$

$$y_B = \sqrt{P_1} h_B (x_A + x_B) + \sqrt{P_1} h_B e_t + \sqrt{\epsilon P_1 + \sum_{k=1}^M P_k \delta_k^2 + N_0} n_B \quad (4)$$

where  $P_1$  is the relay transmitted power. Again,  $\sqrt{P_1} h_A e_t$  and  $\sqrt{P_1} h_B e_t$  can be modeled as zero mean complex Gaussian random variables with variance  $\sigma_t P_1$ . Then, (3) and (4) can be written as

$$y_A = \sqrt{P_1} h_A (x_A + x_B) + \sqrt{(\sigma_t + \epsilon) P_1 + \sum_{j=1}^M P_j \delta_j^2 + N_0} n_A \quad (5)$$

$$y_B = \sqrt{P_1} h_B (x_A + x_B) + \sqrt{(\sigma_t + \epsilon) P_1 + \sum_{k=1}^M P_k \delta_k^2 + N_0} n_B \quad (6)$$

The destination (which is assumed to be source B) subtracts its transmitted signal to get the signal transmitted from the other source. Then, the instantaneous signal to interference ratio (SINR) at the destination node B is given by

$$\gamma = P_1 |h_B|^2 / ((\sigma_t + \epsilon) P_1 + \sum_{k=1}^M P_k \delta_k^2 + N_0)$$

The average SNGR is given by [9]

$$R_t = \gamma |_{\epsilon=0, M=0, \sigma_t=0} - \gamma |_{\epsilon \neq 0, M \neq 0, \sigma_t \neq 0} / E(\gamma |_{\epsilon=0, M=0, \sigma_t=0}) \quad (7)$$

It is clear that  $\gamma |_{\epsilon=0, M=0, \sigma_t=0} = \frac{P_1 |h_B|^2}{\sum_{k=1}^M P_k \delta_k^2 + N_0}$  and its mean is given by  $E(\gamma |_{\epsilon=0, M=0, \sigma_t=0}) = \frac{P_1 \sigma_B^2}{N_0}$ . Then by substitution of these expressions in (7), the SNGR can be written as

$R_t = \frac{|h_B|^2 ((\sigma_t + \epsilon) P_1 + \sum_{k=1}^M P_k \delta_k^2)}{\sigma_B^2 ((\sigma_t + \epsilon) P_1 + \sum_{k=1}^M P_k \delta_k^2 + N_0)}$ . If we use N relays instead of one relay, the destination instructively applies maximal ratio combiner (MRC) to add the signals received from the relays. Then, the total instantaneous signal to noise ratio at the MRC output is given by:

$$\gamma = \sum_{i=1}^N \frac{P_i |h_{Bi}|^2}{(\sigma_t + \epsilon) P_i + \sum_{k=1}^M P_k \delta_k^2 + N_0} \quad (8)$$

where  $P_i$  is transmitted power of the i-th relay and  $h_{Bi}$  is the channel coefficient of from the i-th relay to the destination. Then the SNGR in this case is given

by  $R_t^M = \frac{\sum_{i=1}^N |h_{Bi}|^2 P_i ((\sigma_t + \epsilon) P_i + \sum_{k=1}^M P_k \delta_k^2)}{\sum_{i=1}^N \sigma_B^2 P_i ((\sigma_t + \epsilon) P_i + \sum_{k=1}^M P_k \delta_k^2 + N_0)}$  and its average is given by

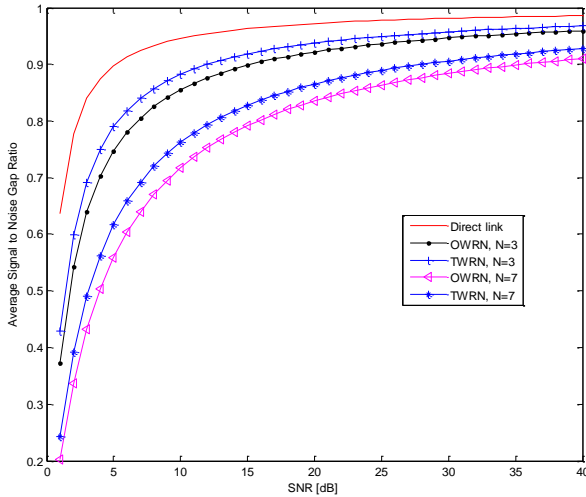


Figure 2 Average SNGR performance comparison of OWRN and TWRN for different values of N and for  $\alpha=0.5$  and  $M=3$

$$E(R_t^M) = \frac{\sum_{i=1}^N \delta_{Bi}^2 P_i (\sigma_t + \epsilon) P_i + \sum_{k=1}^M P_k \delta_k^2}{\sum_{i=1}^N \delta_{Bi}^2 P_i [( \sigma_t + \epsilon) P_i + \sum_{k=1}^M P_k \delta_k^2 + N_o]} \quad (9)$$

### 3.2 Signal to noise gap ratio of one way relaying

In this sub-section the performance of the OWRN in the presence of both the channel estimation error and the co-channel interference is investigated. In the first phase, the signal received at the relay from source A is given by

$$y_r = \sqrt{P_o} (h_A + e_A) x_A + \sum_{z=1}^M \sqrt{P_z} h_z x_z + \sqrt{N_o} n_r \quad (10)$$

where  $n_r$  is a zero mean, unit variance complex Gaussian noise from source to relay. Similar to the derivations of the TWR, (10) can be written as

$$y_r = \sqrt{P_o} h_A x_A + \sqrt{\epsilon P_o + \sum_{z=1}^M P_z \delta_z^2 + N_o} n_r \quad (11)$$

and the received signal at the destination B from the relay after decoding, re-encoding and forwarding, is given by

$$y_B = \sqrt{P_1} h_B \hat{x}_A + \sqrt{\epsilon P_1 + \sum_{z=1}^M P_z \delta_z^2 + N_o} n_B \quad (12)$$

where  $\hat{x}_A = x_A + e_o$  where  $e_o$  is the error of decoding of  $x_A$ . Again,  $\sqrt{P_1} h_{r,d} e_o$  can be modeled as zero mean complex Gaussian random variables with variance  $\sigma_o P_1$  where  $\sigma_o < \sigma_1$ . From (12) the instantaneous SINR at the destination B is given by  $\gamma = P_1 |h_B|^2 / (N_o + \sum_{z=1}^M P_z \delta_z^2 + (\sigma_o + \epsilon) P_1)$ . The SNGR in case of OWRN is given by

$$R_o = \frac{|h_B|^2 (\sum_{z=1}^M P_z \delta_z^2 + \epsilon P_1)}{\delta_B^2 (N_o + \sum_{z=1}^M P_z \delta_z^2 + (\sigma_o + \epsilon) P_1)}$$

For N relays, the SNR gap ratio at the output of MRC is given by

$$R_o^M = \frac{1}{\sum_{i=1}^N P_i \delta_{Bi}^2} \sum_{i=1}^N \frac{P_i |h_{Bi}|^2 (\sum_{z=1}^M P_z \delta_z^2 + \epsilon P_1)}{N_o + \sum_{z=1}^M P_z \delta_z^2 + (\sigma_o + \epsilon) P_1} \quad (13)$$

and its average is given by

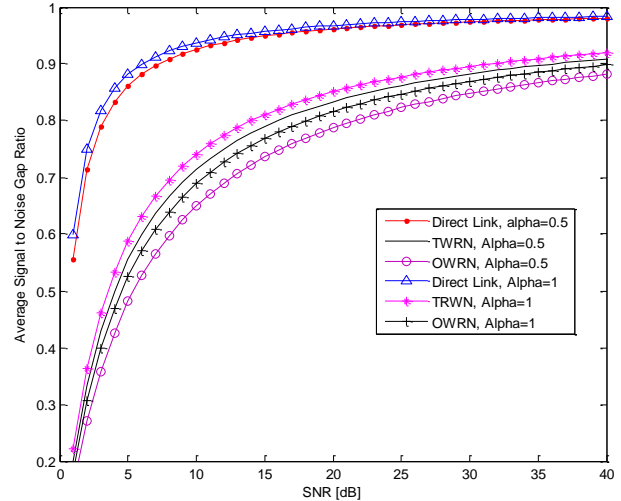


Figure 3 Average SNGR performance comparison of OWRN and TWRN for different M and  $\alpha=0.5$  and  $N=7$

$$E(R_o^M) = \frac{1}{\sum_{i=1}^N P_i \delta_{Bi}^2} \sum_{i=1}^N \frac{P_i \delta_{Bi}^2 (\sum_{z=1}^M P_z \delta_z^2 + \epsilon P_1)}{N_o + \sum_{z=1}^M P_z \delta_z^2 + (\sigma_o + \epsilon) P_1} \quad (14)$$

### 3.3 Signal to noise gap ratio of direct link

For comparison purposes we include the SNR gap ratio of the direct link (i.e. when the relays do not cooperate in communications). The received signal at the destination B from the source is given by

$$y_{s,d} = \sqrt{P_o} (h_{s,d} + e_o) x + \sum_{k=1}^M \sqrt{P_k} h_k x_k + \sqrt{N_o} n_{s,d} \quad (15)$$

where  $e_o$  represents the channel estimation error and the second term represents the co-channel interference. Similar to the above derivations, the instantaneous SINR at the destination is given by  $\gamma_d = P_o |h_{s,d}|^2 / (N_o + \sum_{k=1}^M P_k \delta_k^2 + \epsilon P_o)$  and the average SNGR of the direct link  $R_d$  is given by  $R_d = |h_{s,d}|^2 (\sum_{k=1}^M P_k \delta_k^2 + \epsilon P_o) / \delta_{s,d}^2 (N_o + \sum_{k=1}^M P_k \delta_k^2 + \epsilon P_o)$  and its average is given by

$$E(R_d) = \frac{(\sum_{k=1}^M P_k \delta_k^2 + \epsilon P_o)}{(N_o + \sum_{k=1}^M P_k \delta_k^2 + \epsilon P_o)} \quad (16)$$

## 4 NUMERICAL RESULTS

In this section, numerical results are presented to illustrate the impact of the two-way relaying on the channel estimation error and/or co-channel interference. Power transmission has the largest SNGR compared to the TWRN and OWRN trans-

mission. This implies that if the network is affected by channel estimation error and co-channel interference simultaneously, OWRN and TWRN cooperative transmission is able to reduce their impact more co-channel interference achieved. Moreover, OWRN has smaller SNGR than TWRN.

This is because the decoding error is higher in TWRN than OWRN due to large noise level at the relay which results from the sum of the noises from source A and B at the relay. The effect of the number of interferers on the ability of OWRN and TWRN to reduce the effect of channel estimation error and co-channel interference is illustrated in Fig. 3 for  $\alpha=0.5$  and  $N=7$ . The figure shows that as the number of interferers increases, the average SNGR increases. The effect of the variance of the channel estimation error on the average SNGR is shown in Fig. 4 for  $M=2$ ,  $N=7$ . The results show that as the variance increases, the average SNGR increases.

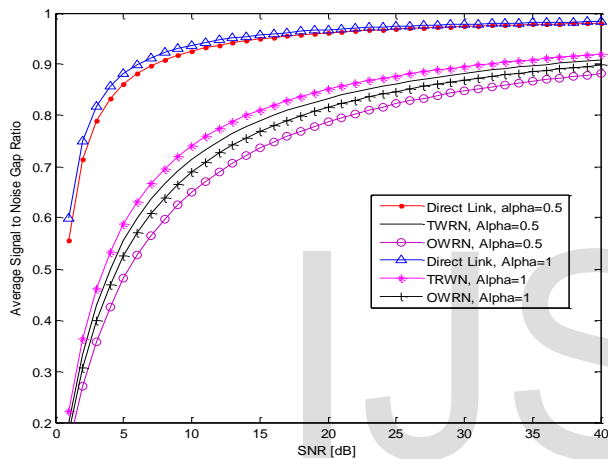


Figure 4. Average SNGR performance comparison of OWRN and TWRN for  $M=2$ ,  $N=7$  and different values of  $\alpha$

## 5 CONCLUSIONS

A performance comparison of TWRN and OWRN decode and forward relaying has been investigated in presence of channel estimation error and co-channel interference. It has been found that, channel estimation error and co-channel interference increases the average SNGR ratio which measures the reduction in SNR due to these errors allocation scheme is considered, in which  $P_0 = P/2$  and  $P_i = P/(2N)$ ,  $i = 1, 2, \dots, N$  and the powers  $P_j$  and  $P_z$  is taken as  $P_j = P_0$  and  $P_z = P_i$ . Fig. 2 shows the average SNGR as a function of  $SNR=P/N_0$  of OWRN, TWRN, and direct link in presence of channel estimation error and co-channel interference. The figure is plotted for  $M=3$ ,  $\epsilon = 0.5$  and different number of relays. The figure shows that direct. Furthermore, it has been concluded that TWRN and OWRN cooperative communications not only enhance the BER performance but also reduce the effect of channel estimation error and/ or co-channel interference. Finally, OWRN has better performance than

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